METHOD–OF–MOMENTS SOLUTION FOR THE POSTS IN A CIRCULAR WAVEGUIDE

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Abstract––A three–dimensional discontinuity problem of a pair of metallic posts with finite diameter in the TE_{11}–mode circular waveguide is solved by method of moments. Unlike the widely used multifilament representation which leads to a slowly converging series, a multi–strip representation is introduced for the post current modeling. Numerical results are compared with experimental data, including those given by the other authors.

Introduction

In circular waveguide, the concentric discontinuities have been studied by many authors, but the studies of non–axisymmetric discontinuities, especially those with finite thickness, are very scarce. The purpose of this paper is to develop a method–of–moments solution for electromagnetic scattering by a pair of perfectly conducting posts in a TE_{11}–mode circular waveguide, as shown in Fig.1, and to compute the parameters of equivalent circuit shown in Fig.2. The difficulty of our problem lies in the absence of coincidence of post surface with the cylindrical coordinate lines so that all possible higher order modes must be taken into account. Multifilament current representation leads to a slowly converging series which is not convenient for computation when the field on the post surface is expanded in terms of waveguide modes. In rectangular waveguide, this series can be converted to a rapidly converging one by introducing certain auxiliary series, as a result, multifilament approximation has been widely used for post, or probe, problems(1),(2). For circular waveguide posts, such an auxiliary series is difficult to find, so we try a new approach to post current modeling and a multi–strip representation is proposed. The true currents on post surfaces are replaced by equivalent planar currents inside the posts in a number of waveguide cross sections, i.e. xy planes, then the unknown planar currents are expanded in the xy planes. By using expansion– and testing functions chosen properly and field matching along generatrix lines of posts, the convergence of the series can be improved to a certain extent so that the numerical calculation goes to feasible.

Formulation of the problem

Consider the case of two strips and four–term expansion. As shown in Fig.3, the equivalent currents on the imaginary strips can be expressed as

$$\vec{J}(x,y) = \begin{cases} \hat{x} \sum_{j=1}^{4} I_j F_j(x,y) & \text{on} \quad \sigma^- \\ \hat{x} \sum_{j=5}^{8} I_j F_j(x,y) & \text{on} \quad \sigma^+ \end{cases} \tag{1}$$

where

$$F_j(x,y) = \psi_j(x)\ \varphi_j(y) \tag{2}$$

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$$\psi_1=\psi_3=\psi_5=\psi_7 \qquad \psi_2=\psi_4=\psi_6=\psi_8$$

$$\varphi_1=\varphi_2=\varphi_5=\varphi_6 \qquad \varphi_3=\varphi_4=\varphi_7=\varphi_8$$

$$\psi_1(x)=\sin kw \tag{3a}$$

$$\psi_2(x)=\alpha\ (1-\cos kw)\ -\sin kw \tag{3b}$$

$$\alpha=\frac{kl-\sin kl\ \cos kl}{(1-\cos kl)^2} \qquad w=x-a+l$$

$$\varphi_1(y)=\frac{1}{b} \qquad \varphi_3(y)=\frac{\pi}{2b}\ \cos\frac{\pi y}{b}-\frac{1}{b} \tag{4}$$

Here a is the waveguide radius, l is the post length and b is the strip width. The constant α makes ψ_1 and ψ_2 orthogonal to one another in the interval $0<w<l$ (2):

$$\int_0^l \psi_1(x)\ \psi_2(x)\ dw=0 \tag{5}$$

Similar orthogonality is held for φ_1 and φ_3 :

$$\int_{-b/2}^{b/2} \varphi_1(y)\ \varphi_3(y)\ dy=0 \tag{6}$$

The strips σ^- and σ^+ must be inside the post of diameter d , so we have

$$0<c<d/2 \tag{7}$$

$$0<b\leqslant(\ d^2-4c^2\ )^{1/2} \tag{8}$$

Six testing generatrix lines are uniformly distributed on the post surface. Symmetry shows that only four lines (1,2,3 and 4) need to be tested. The $\psi_i(x)$ of Eq.(3) are used as the testing functions and the following matrix equation is obtained

$$\mathbf{ZI}=\mathbf{V} \tag{9}$$

where

$$Z_{ij}=-\int_0^{L(y_i)} \vec{A}_j(x,y_i,z_i)\cdot\hat{x}\psi_i(x)\ dw \tag{10}$$

$$V_i=\int_0^{L(y_i)} \vec{E}^{inc}(x,y_i,z_i)\cdot\hat{x}\psi_i(x)\ dw \tag{11}$$

$$\vec{A}_j(x,y,z)\cdot\hat{x}=-\sum_p e^{-\gamma_p|z-z_j|}\ \vec{e}_p(x,y)\cdot\hat{x}\cdot$$

$$\int_{-b/2}^{b/2}\int_0^{L(y')}\vec{e}_p(x',y')\cdot\hat{x}F_j(x',y')\ dwdy' \tag{12}$$

$$L(y)=l-a+(a^2-y^2)^{1/2}$$

The $\vec{e}_p$ is normalized transverse electric field of the p–th mode and γ_p is the propagation constant. It is vitally important to consider all the possible TEnq and TMnq modes in the summation of (12). Careful analysis of field distribution indicates that only the modes with $\cos n\varphi$ variations in radial electric components can be excited when both the posts and the incident TE_{11} mode are x–oriented, and then only the TEnq and TMnq modes with odd index n are needed because the $x=0$ plane is a electric wall. The symmetry of posts about the $z=0$ plane suggests that the even– and odd–excitation can be treated separately so the 8–order matrix equation (9) can be decomposed into two of the 4–order equations, but the condition of the matrix in Eq.(9) has been greatly improved by orthogonality of (5) and (6) and such a decomposition is not important.

Numerical results and discussion

A computer program has been developed to carry out the solution procedure. To test the convergence of the moment solution, the higher order modes in the summation of (12) can be specified. Typical dimensional parameters ($0.2930 < a/\lambda < 0.3827$, $0.02 < d/\lambda < 0.1$, $l/a < 0.5$) are used for trial computations. It is found that TE_{n1}, TE_{n2}, and TM_{n1}, TM_{n2}, TM_{n3} make a significant contribution to the matrix elements. The contribution of the TMnq with $q > 3$ is found to be small, and the TEnq with $q > 2$ can be ignored. Summation over modes corresponding to black blocks in Fig.4 gives a certain accurate values of circuit parameters Xa and Xb with the unitary–condition error of scattering matrix less than 0.001, while the white blocks correspond to the first 700 modes given by Beattie(3). If all modes with $n > 29$ are neglected, typical truncation error for Xa is less than 0.03.

The equivalent circuit parameters are insensitive to the location c and the width b of strips except the values close to the limits of (7) or (8). The input VSWR of post-loaded circular waveguide with $a=26.9$mm, $l=9.11$mm and $d=4.0$mm has been tested and calculated. Both the experimental and theoretical results are shown in Fig.5. A comparison between numerical results and measured data given by Ishida et.al. (4) for the post susceptance B ($=-1/$ Xa) is given in Fig.6. Calculated values of Xa and Xb for $a/\lambda=0.321$ and 0.355 are shown in Fig.7.

The results obtained in this paper can be used to analysis of circular waveguide devices such as polarizers and filters. The multi-strip current modeling should prove useful in solving a variety of circular waveguide discontinuities with finite thickness and without axisymmetry.

References

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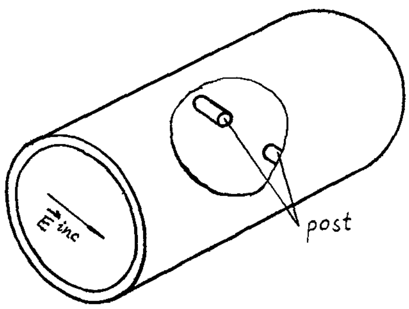

Fig.1 A pair of posts in circular waveguide

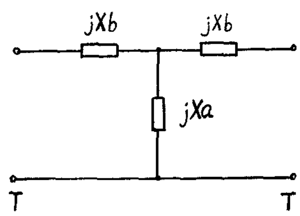

Fig.2 Equivalent circuit for posts

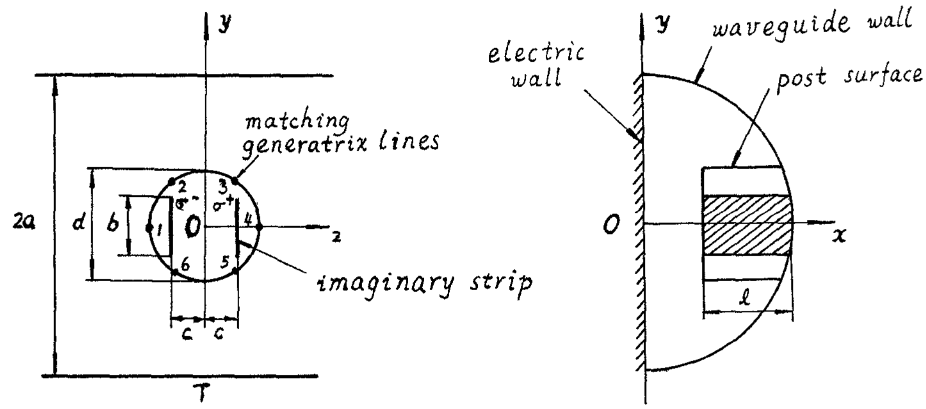

Fig.3 Multi-strip current modeling

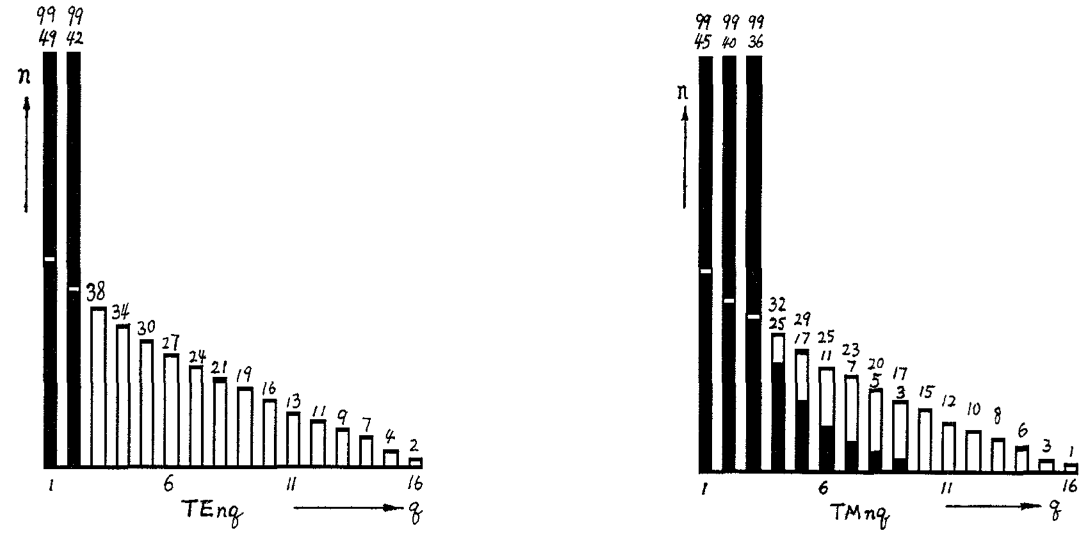

Fig.4 Higher order modes considered in summation of (12)

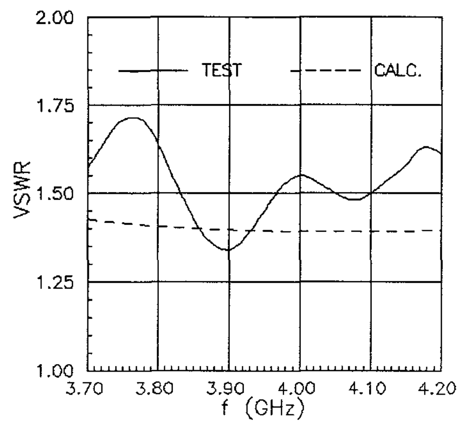

Fig.5 Variation of VSWR versus frequency

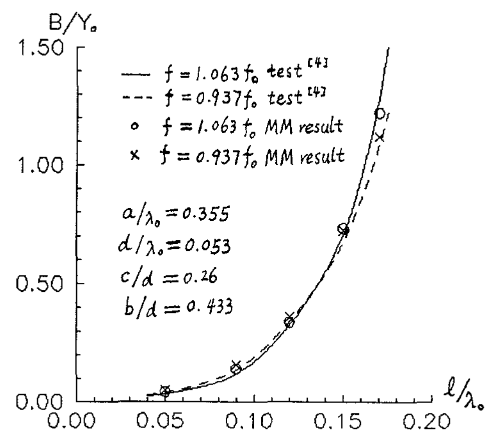

Fig.6 Equivalent susceptance of posts

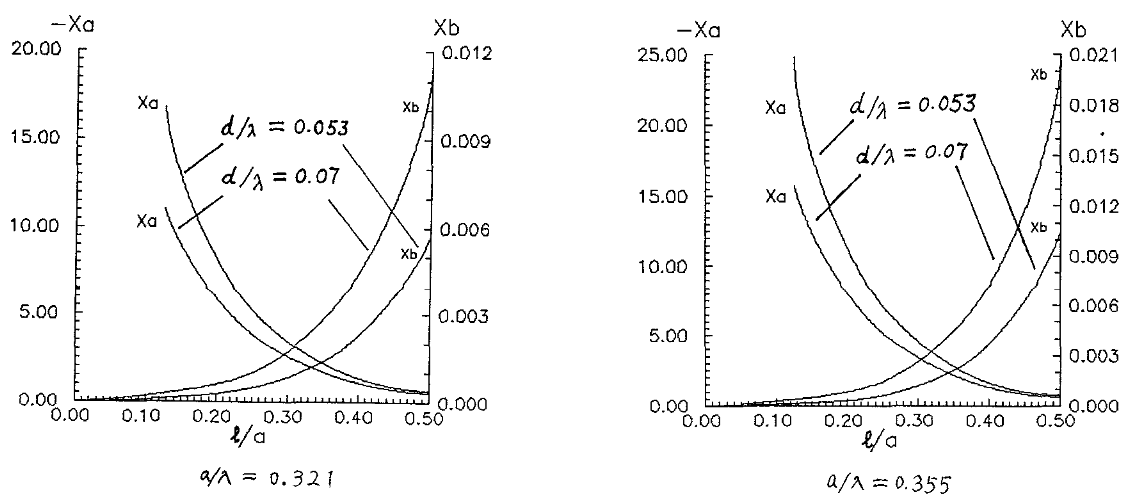

Fig.7 Calculated values of circuit parameters (c/d=0.26, b/d=0.433)